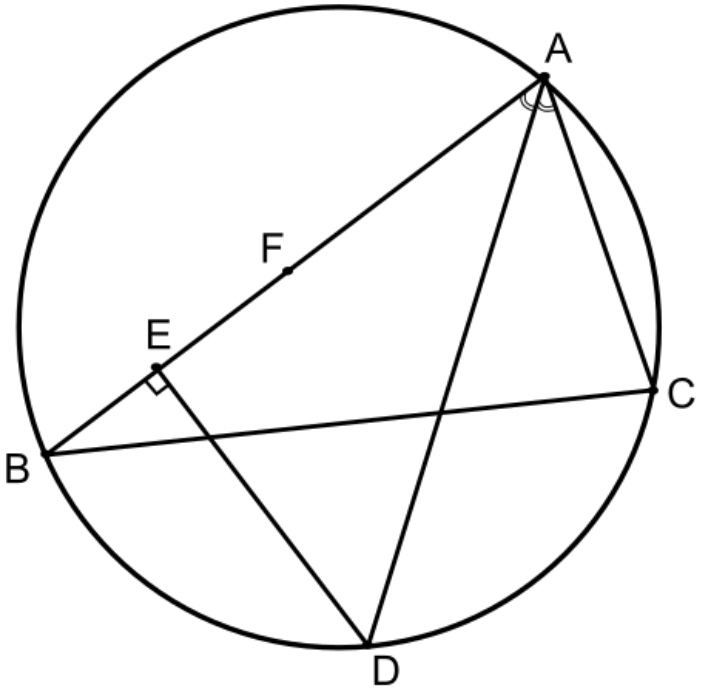
Cash Award Question of July-2025



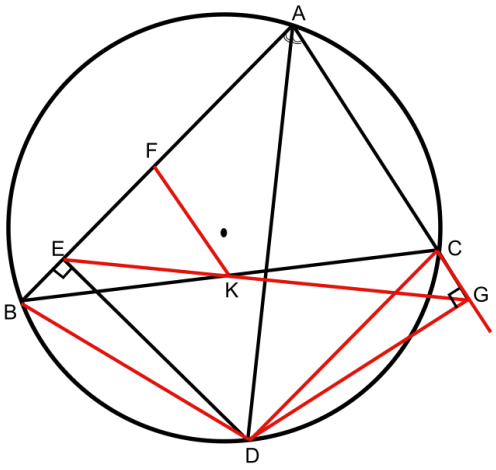
In the picture, ΔABC is inscribed in the circle. The angle bisector of $\angle BAC$ meets the circle at D. DE $\perp$ AB is drawn. F is the midpoint of AB.

Prove: AC = 2EF.

*****

Question framed by
DR. M. RAJA CLIMAX, IRS
Asst. Commissioner of Customs & GST (Rtd),
Madurai, Tamil Nadu, India

Author's Solution July-2025



Given :

ΔABC is inscribed in the circle. The angle bisector of ∠BAC meets the circle at D. DE ⊥ AB. F is the midpoint of AB.

To Prove:

AC = 2EF

Construction:

Draw DG⊥AC produced.

Join DC, DB & EG. Let EG & BC intersect at K. Join FK.

Proof:

AD is the bisector of $\angle A$

$\Rightarrow DC = DB$ ----------(1)

also $\Rightarrow DE = DG$ ----------(2) ($\because$ $Perpendicular\ from\ any\ point\ on\ the\ angle\ bisector\ to\ the\ sides\ of\ the\ angle\ will\ be\ equal$)

(1) & (2) →

$\Delta DEB \cong \Delta DGC$ (RHS Principle)

⇒ EB=CG ----------(3)

(2) → $\Delta ADE \cong \Delta ADG$ (RHS principle & AD is common)

$\Rightarrow AE = AG$ ----------(4)

$\Rightarrow \angle AEG = \angle AGE$ ----------(5)

EKG is a transversal of ΔABC

$\Rightarrow \frac{AE}{EB} \times \frac{BK}{KC} \times \frac{CG}{GA} = 1$ (Menelaus Theorem) --------- (6)

(3), (4) & (6) ⇒ BK = KC & K is the midpoint of BC.

⇒ FK ‖ AC ($\because$ $F\ is\ the\ midpoint\ of\ AB - given$) ----------(7)

⇒ 2FK = AC ---------- (8)

(5) & (7) →

$\angle FEK = \angle FKE$

$\Rightarrow EF = FK$ ----------(9)

(8)& (9) →

AC = 2EF ---------- Proved.

****

Solution given by
DR. M. RAJA CLIMAX, IRS
Asst. Commissioner of Customs & GST (Rtd),
Madurai, Tamil Nadu, India